QUANTIZATION IN ALGEBRA AND HOMOTOPY THEORY

SUMMER 2021

Summary

The notion of *quantization* has taken on a life of its own in mathematics in the past few decades: the concept plays a crucial role in many areas, including algebraic geometry, representation theory, and differential geometry. Our goal will be to study quantization in its various guises, with an emphasis on structural aspects as well as key explicit examples. In particular, we will discuss geometric quantization of symplectic manifolds and deformation quantization of Poisson algebras. Our focus will mainly be on the latter, where we will also study generalizations to characteristic p and describe an interpretation via cyclotomic spectra.

A lot of our discussion will be motivated by algebro-topological and algebro-geometric considerations, so familiarity with algebraic geometry and algebraic topology (such as derived categories of quasicoherent sheaves, spectra, ...) will be very useful. Background in physics won't be necessary (since our emphasis will be on mathematical aspects), but it might be useful for motivation.

Topics

These are subject to change depending on interest.

- Background on classical and quantum field theory, esp. focusing on (mathematical) examples like Chern-Simons and Yang-Mills theory, and σ-models.
- Deformation quantization, esp. Kontsevich's existence theorem: proof via the formality of the little disks operad.
- (Framed) little disks operads and the BV-formalism, and holomorphic analogue of Kontsevich's theorem à la Bogomolov-Tian-Todorov.
- Frobenius-constant quantizations in characteristic p à la Bezrukavnikov-Kaledin.
- "Koszul dual" to deformation quantization via circle actions (Hochschild-Kostant-Rosenberg, Kontsevich-Soibelman) and the Ω-deformation (à la Beem-Ben-Zvi-Bullimore-Dimofte-Neitzke and Butson).
- "Koszul dual" to Frobenius-constant quantizations via cyclotomic spectra.
- Examples via Coulomb branches (à la Braverman-Finkelberg-Nakajima) and Nekrasov's Ω-deformation.
- Geometric quantization, esp. for Chern-Simons theory (relation to the determinant line bundle on Bun_G and Wess-Zumino-Witten conformal field theory).

Potential other topics

- Supersymmetry, e.g., Witten's interpretation of Morse theory.
- Proof of Kontsevich's theorem via the path integral of a 2-dimensional σ-model.
- Costello-Gwilliam's approach to BV-quantization in quantum field theory via factorization algebras.

Logistics

We will probably start in the beginning of June. My hope is that we will be able to meet in-person, but it is likely that there will regardless be a Zoom component. I will try to type up notes before each meeting.

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